Section (10)
"2/4/2015"

1 DC RLC

$$\alpha = -R/2L$$
$$\beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{LC}\right)}$$

case I if $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$ overdamped

$\therefore\ i = C_1 e^{(\alpha+\beta)t} + C_2 e^{(\alpha-\beta)t}$

or $\boxed{i_1 = e^{\alpha t}\left(C_1 e^{\beta t} + C_2 e^{-\beta t}\right)}$ $\Rightarrow$ $D_1 = \alpha + \beta$, $D_2 = \alpha - \beta$

case II if $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$ critically damped

$$\boxed{i_2 = e^{\alpha t}(C_1 + C_2 t)}$$

case III if $\left(\frac{R}{2L}\right)^2 \cancel{>} \frac{1}{LC}$ underdamped

$$\boxed{i_3 = e^{\alpha t}(C_1 \cos\beta t + C_2 \sin\beta t)}$$

at $t=0 \rightarrow i=0$
نعوض في i
KVL نحتاج معادلة
i_2 , $\frac{di}{dt} = ?$
لإيجاد الثوابت

2 AC RLC

Case I overdamped $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$

$$i_1 = \underbrace{e^{\alpha t}\left(C_1 e^{\beta t} + C_2 e^{-\beta t}\right)}_{I_c \text{ نفس الـ dc}} + \underbrace{\frac{V_{max}}{\sqrt{(R^2) + \left(\frac{1}{\omega C} - \omega L\right)^2}} \sin\left(\omega t + \phi + \tan^{-1}\left(\frac{\frac{1}{\omega C} - \omega L}{R}\right)\right)}_{I_p \text{ زي ما هو}}$$

Case II
$\hookrightarrow$ critically damped $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$

$$i_2 = e^{\alpha t}(C_1 + C_2 t) + I_p$$

Case III underdamped $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$

$$i_3 = e^{\alpha t}(C_1 \cos\beta t + C_2 \sin\beta t) + I_p$$

3 AC-RL

$i = i_p + i_c \longrightarrow D + ? = 0$

$i_c = C e^{mt}$

$$i_p = \frac{V_{max}}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\omega t + \phi - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

at $t=0 \longrightarrow i=0$
~~[illegible]~~ i_t نعوض في
~~KVL [illegible]~~

4 AC-RC

$i = i_p + i_c \longrightarrow D + ? = 0$

$i_c = C e^{mt}$

$$i_p = \frac{V_{max}}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \sin\left(\omega t + \phi + \tan^{-1}\left(\frac{1}{\omega RC}\right)\right)$$

at $t=0 \rightarrow i = ?!$ من معادلة KVL
& ~~t~~ $t=0$ ثم نعوض في i في معادلة i_t لإيجاد الثوابت

Sheet (8)

1 A series RLC with $R = 3000\,\Omega$, $L = 10h$, $C = 200\mu F$ has a constant $V = 50V$ applied at $t = 0$. Find the Current Transient and the maximum value of The current if the Capacitor has no initial charge.

— Solution —

$$50 = Ri + L\frac{di}{dt} + \frac{1}{c}\int i\,dt$$

$$\boxed{50 = 3000i + 10\frac{di}{dt} + \frac{1}{200\times10^{-6}}\int i\,dt} \quad (1)$$ بتفاضل الطرفين

or $(D^2 + 300D + 500)\,i = 0$ بحل المعادلة التربيعية بالقانون العام

$$D = \frac{-B \mp \sqrt{B^2 - 4AC}}{2A} = \alpha \mp \beta$$

Note $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$

$\therefore D_1 = -298.3$ $(\alpha+\beta)$, $D_2 = -1.67$ $(\alpha-\beta)$

$$\therefore \boxed{i = C_1 e^{-0.167t} + C_2 e^{-298.3t}} \quad (2)$$

to Find C_1, C_2

at $t = 0, 0^+ \rightarrow i = 0$ sub in 1, 2

$$\therefore \boxed{0 = C_1 + C_2} \rightarrow (3)$$

and at $i = 0 \rightarrow$ $50 = 10\,di/dt$ or $di/dt = 5 \rightarrow (4)$

at $t = 0$ $\leftarrow$ بـ $di/dt = 5$ $\therefore$ نفاضل المعادلة 2 ونعوض بـ 5

$$5 = (-0.167)C_1 e^{-0.167t} - (298.3)\,C_2 e^{-298.3t}$$

$$\boxed{5 = -0.167\,C_1 - 298.3\,C_2} \rightarrow (6)$$

بحل المعادلة ٣ ، 6 نحصل $C_1 = 0.168$ ، $C_2 = -0.0168$

$$\therefore \boxed{i = 0.0168\,e^{-1.67t} - 0.0168\,e^{-298t}}$$ مطلوب

to Find max Current $\therefore$ at $\frac{di}{dt} = 0$ (القيم العظمى)

or $(0.0168)(-1.67)\,e^{-1.67t} - (0.0168)(-298.3)\,e^{-298t} = 0$

$\hookrightarrow t = 0.0175$ sec

[2] A series RL circuit with $R = 50\,\Omega$ and $L = 0.2\,H$ has a sinusoidal voltage source $v = 150\sin(500t + \phi)$ applied at a time when $\phi = 0$ find complete current.

— Sol —

$$Ri + L\,di/dt = v$$

$$50i + 0.2\,di/dt = 150\sin 500t$$

$$\therefore (D + 250)i = 750\sin 500t$$

→ ~~complementary sol. $i = C e^{-250t}$~~

use Final equation $= C e^{-250t} + i_p$

$$i_p = \frac{V_{max}}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\omega t + \phi - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

$$= \frac{150}{\sqrt{(50)^2 + (500)^2(0.2)^2}} \sin\left(500t + 0 - \tan^{-1}\left(\frac{500 \times 0.2}{50}\right)\right)$$

$$i_{total} = C e^{-250t} + 1.34\sin(500t - 63.4^\circ)$$

at $t = 0$ $i = 0$

$$\therefore 0 = C + 1.34\sin(-63.4)$$

$$\therefore C = 1.2$$

$$\boxed{\therefore i_t = 1.2 e^{-250t} + 1.34\sin(500t - 63.4^\circ)}$$

3] series RC with $R = 100$, $C = 25\,\mu F$, sinusoidal voltage $250 \sin(500t + \phi)$, applied at a time when $\phi = 0$, Find I (no initial voltage on cap)

Sol

$$\overset{R}{100}\, i + \frac{1}{C}\int i\,dt = 250 \sin 500t$$

OR $\cong$

$$(\Phi + 400)\, i = 1250 \cos(500t)$$ بعد التفاضل

Sol

$$i = C\, e^{-400t} + \frac{V_{max}}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \sin\left(\omega t \ast \phi + \tan^{-1}\left(\frac{1}{\omega R C}\right)\right)$$

$$\boxed{i = C \cdot e^{-400t} + 1.955 \sin(500t + 38.7^\circ)}$$

at $t = 0 \longrightarrow i = \frac{250}{100} \sin 0 = 2.5$

نعوض في $0 = i(0)$ at

$\therefore\ 2.5 = C\, e^{-400 \times 0} + 1.955 \sin(0 + 38.7)$

$\therefore\ C = -1.22$

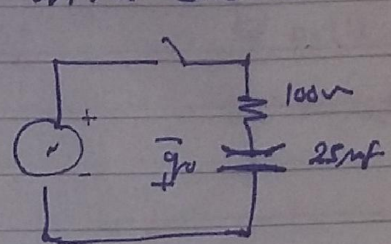
$$\therefore\ \boxed{i = -1.22\, e^{-400t} + 1.955 \sin(500t + 38.7^\circ)}$$

$\boxed{4}$ in RC circit shown $v = 250\sin(500t + \phi)$

at $\phi = 45°$ → switch closed , initial charge

$q_0 = 5000 \times 10^{-6}$ Col. on capacitor with shown

Polarity , Find I



Sol

∴ نفس قيم المطلب السابق

$$i = c\,e^{-400t} + 1.955\sin(500t + \phi + 38.7)$$

$$= c\,e^{-400t} + 1.955\sin(500t + 83.7°)$$

at $t = 0$ $\quad i = \dfrac{v_{in} + V_c}{100} = \dfrac{(250\sin 45) + (q_0/C)}{100}$ → 200V

$$i = (250\sin 450 + 200)/100 = 3.77$$

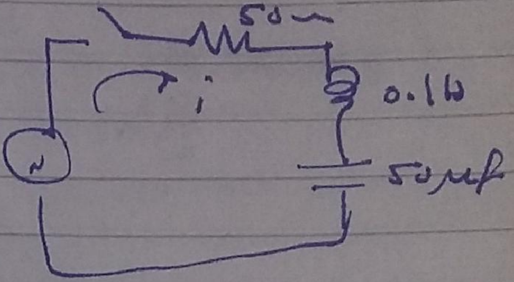
∴ at $t = 0$ → $i = 3.77A$

$$3.77 = c\,e^{0} + 1.955\sin(0 + 83.7°)$$

$$c = 1.83$$

$$\therefore \boxed{i = 1.83\,e^{-400t} + 1.955\sin(500t + 83.7°)}$$

5] Series RLC has source $100\sin(1000t+\phi)$, if switch closed :: $\phi = 90$, find I if no initial voltage on cap



Sol.

$$50i + 0.1\frac{di}{dt} + \frac{1}{50\times10^{-6}}\int i\,dt$$

$$= 100\sin(1000t+\phi) \rightarrow (1)$$

تفاضل الطرفين تفاضل كلي

$$(D^2 + 500D + 2\times10^5)\,i = 10^6\cos(1000t+90^\circ) \rightarrow I$$

$$D_1 = \underset{\alpha}{-250} + \underset{j\beta}{j371} \qquad D_2 = -250 - j371 \quad \alpha - j\beta$$

$$i_c = e^{\alpha t}[C_1\cos\beta t + C_2\sin\beta t] \qquad \left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$$

$$\boxed{i_c = e^{-250t}[C_1\cos 371t + C_2\sin 371t]} \rightarrow II$$

$$\boxed{i_p = \frac{V_{max}}{\sqrt{R^2+\left(\frac{1}{wc}-wL\right)^2}}\sin\left[wt+\phi+\tan^{-1}\frac{\left(\frac{1}{wc}-wL\right)}{R}\right]} \rightarrow III$$

$$i_p = 1.06\sin(1000t+32) \rightarrow III$$

$$\therefore i = e^{-250t}[C_1\cos 371t + C_2\sin 371t] + 1.06\sin(1000t+32)$$

at $t=0 \rightarrow i_0 = 0$ & eq (I) :: $0.1\,di/dt = 100\sin 90$

$\therefore di/dt = 1000$

&

$$\boxed{0 = e^0[C_1\cos 0 + C_2\sin 0] + 1.06\sin 32}$$

نعوض بها

$$di/dt = (1.06)(1000)\cos(1000t+32) +$$

$$-250e^{-250t}[C_1\cos 371t + C_2\sin 371t] + e^{-250t}[-371C_1\sin 371t + 371C_2\cos 371t]$$

ثم نعوض عن مشتقة $di/dt = 1000$ ونعوض عن C_1 بالأرقام C_2

$$\boxed{C_1 = -0.562} \rightarrow \boxed{C_2 = -0.104}$$

$$\therefore \boxed{i = e^{-250t}\left[-0.562 \cos 371t - 0.104 \sin 371t\right) + 1.06 \sin(1000t + 32^\circ)\right]}$$

(خلاصة الفكرة)

1- underdamped — خلاصة المسألة — الشرط

$D_1 = \alpha + \beta$ $\quad D_2 = \alpha - \beta$ $\quad \therefore \left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$

$$i_t = e^{\alpha t}\left[C_1 \cos\beta t + C_2 \sin\beta t\right] + \underbrace{\frac{V_{max}}{\sqrt{R^2 + \left(\frac{1}{wc} - wL\right)^2}} \sin\left[wt + \phi + \tan^{-1}\left(\frac{\frac{1}{wc} - wL}{R}\right)\right]}_{I_p}$$

2- if $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$ critically damped

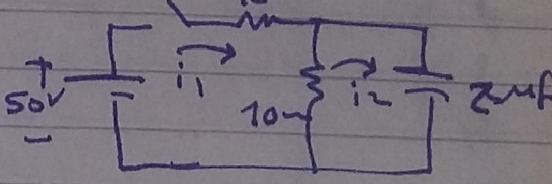
$$i_{total} = e^{\alpha t}\left[C_1 + C_2 t\right] + \underbrace{\frac{V_{max}}{\sqrt{R^2 + \left(\frac{1}{wc} - wL\right)^2}} \sin\left(wt + \phi + \tan^{-1}\left(\frac{\frac{1}{wc} - wL}{R}\right)\right)}_{I_p}$$

3- if $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$ overdamped

$$\therefore i_{total} = e^{\alpha t}\left[C_1 e^{\beta t} + C_2 e^{-\beta t}\right] + I_p$$

$\left(\frac{R}{2L}\right)^2 \quad \frac{1}{LC}$ تختلف حسب I_c ، ثابتة I_p.

(6) in two mesh shown in fig, switch closed at $t=0$
Find transient mesh current i_1 and i_2 shown and V_c



Sol

at S closed

loop 1 $\quad 50 = 20 i_1 - 10 i_2$

$\therefore 0 = 20 D i_1 - 10 D i_2$ بتفاضل الطرفين

~~loop 2~~ or $\boxed{2 D i_1 = D i_2} \rightarrow (1)$

Loop 2 $\quad 0 = 10 i_2 - 10 i_1 + \frac{1}{C} \int i_2 \, dt$

بتفاضل الطرفين

$0 = 10 D i_2 - 10 D i_1 + \frac{1}{C} i_2$

or $\quad - D i_1 + i_2 \left(D + \frac{1}{10 \underbrace{C}_{2\times10^{-6}}}\right) = 0 \rightarrow \cancel{(2)}$

$- D i_1 + i_2 (D + 5\times10^{4}) = 0 \rightarrow (2)$

بالتعويض بالمعادلة (1) في 2

$\therefore - \frac{D i_2}{2} + (D + 5\times10^{4}) i_2 = 0$

or $\quad (D + 10^{5}) i_2 = 0$

$\therefore$ sol $\quad i_2 = A e^{mt} = A e^{-10^{5} t}$

at $t=0 \longrightarrow$ From eq 2 $\quad \therefore 0 = 10 i_2 - 10 i_1$

$\therefore i_1 = i_2$

From (1) $\quad 50 = 20 i_1 - 10 i_2 = 10 i_2 = 10 i_1$ والتعويض في i_1

$\therefore i_1 = i_2 = 5$

$\therefore$ at $t=0 \quad i_2 = 5 = A e^{0} \quad \therefore A = 5$

or $\quad \boxed{i_2 = 5 e^{-10^{5} t}}$

بالتعويض عن i_2 في المعادلة

$50 = 20 i_1 - 10 \times 5 e^{-10^{5} t}$

or $\quad \boxed{i_1 = \frac{5}{2} + \frac{5}{2} e^{-10^{5} t}} \rightarrow$

$V_c = \frac{1}{C} \int i_2 \, dt$

تكامل بعد تعويض i_2

$V_c = 25 (1 - e^{-10^{5} t})$

① RLC (DC) $\longrightarrow$ $\alpha = -\frac{R}{2L}$, $\beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{LC}\right)}$

if $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$ overdamped

$i = e^{\alpha t}\left(c_1 e^{\beta t} + c_2 e^{-\beta t}\right)$

or $c_1 e^{(\alpha+\beta)t} + c_2 e^{(\alpha-\beta)t}$

$= i_c$

$\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$ critically damped

$i = e^{\alpha t}(c_1 + c_2 t)$

$= i_c$

$\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$ underdamped

$i = e^{\alpha t}\left[c_1 \cos\beta t + c_2 \sin\beta t\right)$

$= i_c$

② RLC (AC) ، $i_c =$ من قوانين ال dc

$i = i_c + i_p$

$i = i_c + i_p$, $i_c =$ من قوانين ال dc

$i = i_c + i_p$, $i_c =$ من قوانين ال dc

[illegible] $I_p = \frac{V_{max}}{\sqrt{(R)^2 + \left(\frac{1}{\omega c} - \omega L\right)^2}} \sin\left(\omega t + \phi + \tan^{-1}\left(\frac{\frac{1}{\omega c} - \omega L}{R}\right)\right)$

at $t=0 \rightarrow i \rightarrow$ calc. from KVL (original equation) to find di/dt $\rightarrow$ then sub in I_t to calc const.

③ RL (AC)

$i_t = i_p + i_c$

$i_c = C e^{mt}$ $\rightarrow$ $D + ? = 0$

$i_p = \frac{V_{max}}{\sqrt{R^2 + (\omega L)^2}} \sin\left(\omega t + \phi - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$

at $t=0 \rightarrow i=0$

ونعوض في معادلة i_t لإيجاد الثوابت

④ RC (AC)

$i_t = i_p + i_c$

$i_c = C e^{mt}$ $\rightarrow$ $D + ? = 0$

$i_p = \frac{V_{max}}{\sqrt{R^2 + \left(\frac{1}{\omega c}\right)^2}} \sin\left(\omega t + \phi + \tan^{-1}\left(\frac{1}{\omega R c}\right)\right)$

at $t=0 \rightarrow i =$ نجيب من معادلة ال KVL الأصلية

ثم نعوض في معادلة I_t لإيجاد الثوابت